Marvelous Mendel's Mathematics

So, a while back (about 130 years ago,) an Abbot by the name of Gregor Mendel was interested in plants and inheritance. For a very long time, humans had known that if they took two individuals with desirable traits and mated them to each other, then they would frequently conceive offspring that also had the desired trait. For example, if you took a corn plant that had produced more kernels and bred it (or crossed it) with a similar plant, then the resulting seeds would grow into plants that were very likely to produce large numbers of kernels. Or, if you crossed a long bodied dog with another, then the offspring would also have long bodies. If you do this long enough, you can artificially select organisms to create new types. In this way, Natives of the American Southwest managed to breed teosinte grass over several generations to create corn. German nobles were able to breed dogs together in order to create a new breed designed to be able to chase badgers and other game down narrow holes: dachshunds were born.

As was fairly typical for the time, Mendel was from a reasonably wealthy family, and was well educated. As an Abbot, he was in charge of a Monastery, where the Catholic monks would spend much of their time farming, caring for their lands, and praying. He had plenty of time to do research.

Mendel spent much of his time examining the patterns in how simple binary traits were inherited from generation to generation. He quickly realized that some traits would predictably seem to disappear in one generation, only to reappear in the next. This would be the first documented instance of a scientist realizing the existence of recessive traits. Better still, he was tracking the breeding of nearly a thousand different pea plants, and there were clear patterns in what he saw.

Much later, we not only know all about Mendel's experiments (and regard him as the Father of Genetics) but we also understand the existence of DNA and the processes of mitosis and meiosis, which he knew nothing about. Best of all, we can explain why he saw the patterns he did in his experiments.

Let us start at the beginning. Mendel began his experiments with populations of pea plants that were true breeding for a trait. What this means is that he had a group of plants that had different alleles for height: short and tall. Whenever he bred short plants with short plants, they would breed true, always producing short offspring without fail. Tall bred with tall would also produce no surprises: only tall offspring would be produced. The question was, what traits would result if he crossed them together?

Well, from our modern perspective, we know a few things already. The true breeding populations were true breeding because the plants were homozygous. A true breeding tall plant could only produce gametes with the tall allele for height (T.) A true breeding short plant could only produce gametes with the short allele for height (t.) When Mendel crossed these plants, he was startled by what he saw. Every single one of the offspring from the cross had the tall phenotype; the short version of the height trait had disappeared entirely! Weirdly, Tall(T) x short(t) = Tall(T).

In retrospect, we know this was because the tall trait was dominant. What had happened was this:

1. The tall plants created gametes through meiosis, each containing the tall allele T.

2. The short plants created gametes through meiosis, each containing the short allele t.

3. When these gametes fused to create zygotes, all the resulting zygotes were heterozygous, with a genotype of Tt. Since tall is the dominant allele for the height trait, all the offspring had the tall phenotype.

Hopefully, all this should be fairly obvious. What is less obvious is what happened when Mendel took those heterozygous offspring and crossed them with each other. Before we get into that, let's introduce a simple way to visualize crosses and calculate genotypes and phenotypes. It is known (after its creator) as a Punnett square. We'll use Mendel's first cross between true breeding (homozygous) tall and short plants.

(As a brief aside, it is worth pointing out that to be a great scientist is to achieve a sort of celebrity that few other professions can match. Sure, movie stars, sports heroes and musicians might be more famous right now, but within a few decades almost none of them will still be famous. You probably wouldn't even recognize any of the famous celebrities of 1950's Hollywood, football, or music, but every single student who reaches 9th grade will have heard of and probably recognize names like Punnett, Mendel, Einstein, Newton, Darwin, and even Watson and Crick. Celebrity is fleeting, but scientific fame belongs to the ages...)

Punnett visualized a simple geometric way to show probabilities that could be produced algebraically. For a simple cross, the method is this:

1. For each parent, record the different gametes that could be produced. For Mendel's first cross, the possible gametes were Tall (T) and Tall (T) from the tall phenotype true breeding parent, and Short (t) and Short (t) from the short phenotype true breeding parent.

|  |  |  |
| --- | --- | --- |
| **2**  TT  tt | T | T |
| t |  |  |
| t |  |  |

2. Then, set up a square with the gametes from one parent along the top, and the gametes from the other parent along the side:

|  |  |  |
| --- | --- | --- |
| **3**  TT  tt | T | T |
| t | T | T |
| t | T | T |

3. Next, carry the alleles from the top row down into the empty squares.

|  |  |  |
| --- | --- | --- |
| **4**  TT  tt | T | T |
| t | Tt | Tt |
| t | Tt | Tt |

4. Then, bring the alleles from the left row across into the middle squares.

Finally, use the genotypes in the middle four squares to calculate the genotypes and phenotypes. In this case, 1:1 (or 100%) of the genotypes are heterozygous (Tt), and 1:1 of the phenotypes are tall because they have the dominant tall (T) allele.

Mendel's first cross was fairly simple. Really, we didn't really need to make a Punnett Square in order to realize that all the offspring would have identical genotypes and phenotypes. His second cross is less obvious, though. By breeding the heterozygous individuals resulting from his first generation, what types of offspring might result?

|  |  |  |
| --- | --- | --- |
| Tt  Tt | T | t |
| T |  |  |
| t |  |  |

I'll start you out by pointing out that since each possible parent is heterozygous, each parent can contribute a gamete with the T allele or the t allele, so the Punnett Square should be set up like so:

Now, it is up to you to complete the cross. Go ahead and carry the alleles down from the top and over from the left side in order to see what combinations are possible. (Use the Square on the right.)

Don't worry. I will wait.

Dum, dum, dee dee deedle deedle daaaa.

(Whistling in the background while you work.)

Okay, once you've completed the square, now look at it. How many different genotypes are there? How many phenotypes? What are the ratios for each when compared to the whole?

If you filled out that last square properly, then you should have gotten three different genotypes. There should have been 1 TT out of the four (1:4 TT), 2 Tt out of the four (2:4 Tt), and 1 tt out of the four (1:4 tt). This means that three out of the four possible combinations would have the tall phenotype, because three of them carried the dominant allele. The ratio with the recessive phenotype of shortness would only be 1:4.

So, Mendel saw that in his next generation, the shortness allele for height had reappeared, but only occasionally. In fact, you tell me. If the recessive trait appears at a 1:4 ratio, what percent of the offspring should have the recessive condition? The answer is in the footnote, but only check after you've tried. What's the point, if not to learn?

So, you've seen the results of a homozygous dominant crossed with a homozygous recessive, and you've seen the results of a heterozygous cross. Take the time now to write out the crosses between a homozygous dominant (TT) and a heterozygote (Tt) and between a homozygous recessive (tt) and a heterozygote (Tt).